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Factorization Model

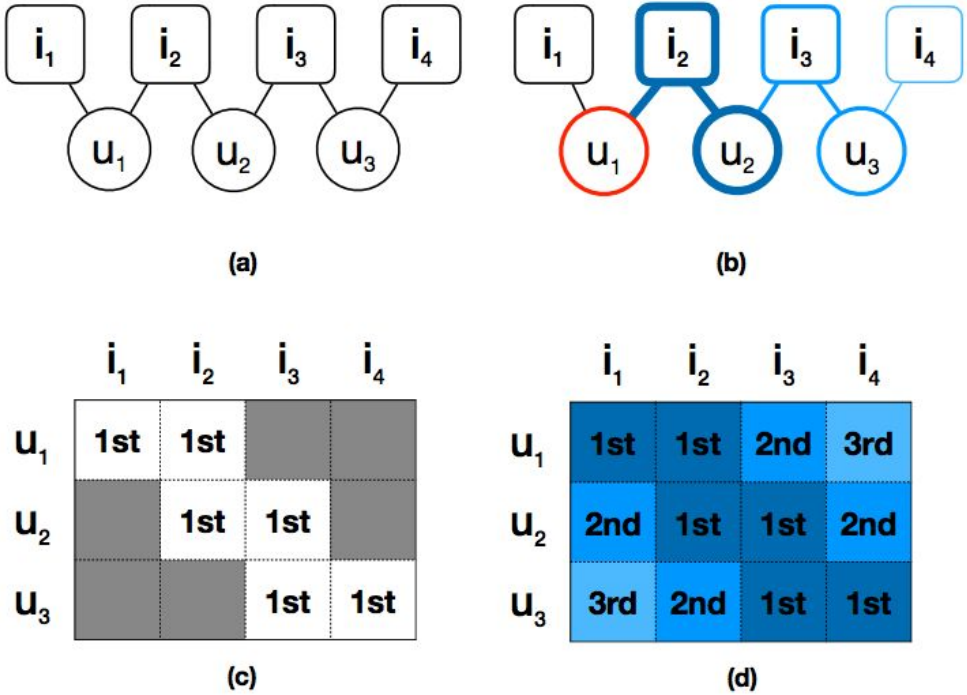
$$\mathcal{L}_{\mathrm{MF}} = \sum_{u,i} c_{ui}\left(a_{ui} - \theta_u^{\mathsf{T}}\theta_i\right)^2 + \lambda_{\Theta}\left\|\Theta\right\|_2^2$$

Ranking-based model

$$\mathcal{L}_{\text{rank}} = \sum_{u,(i,i')} \mathcal{F}\left(\theta_u^{\mathsf{T}}\theta_{i'}, \theta_u^{\mathsf{T}}\theta_i\right) + \lambda_\Theta \|\Theta\|_2^2$$

$$\mathcal{F}(\theta_u^{\mathsf{T}}\theta_{i'}, \theta_u^{\mathsf{T}}\theta_i) = \mathbb{1}_{\{\theta_u^{\mathsf{T}}\theta_{i'} - \theta_u^{\mathsf{T}}\theta_i > \epsilon_k\}} \log\left[\sigma\left(\theta_u^{\mathsf{T}}\theta_{i'} - \theta_u^{\mathsf{T}}\theta_i\right)\right]$$

High-order proximity between users and items within observed interactions



HOP-Rec

$$\mathcal{L}_{HOP} = \sum_{\substack{1 \le k \le K \\ u,(i,i')}} \overbrace{C(k)\,\mathbb{E}_{\substack{i \sim P_u^k \\ i' \sim P_N}}}^{\text{graph model}} \overbrace{\left[\mathcal{F}\left(\theta_u^\intercal \theta_{i'}, \theta_u^\intercal \theta_i\right)\right]}^{\text{factorization model}} + \lambda_\Theta \|\Theta\|_2^2$$